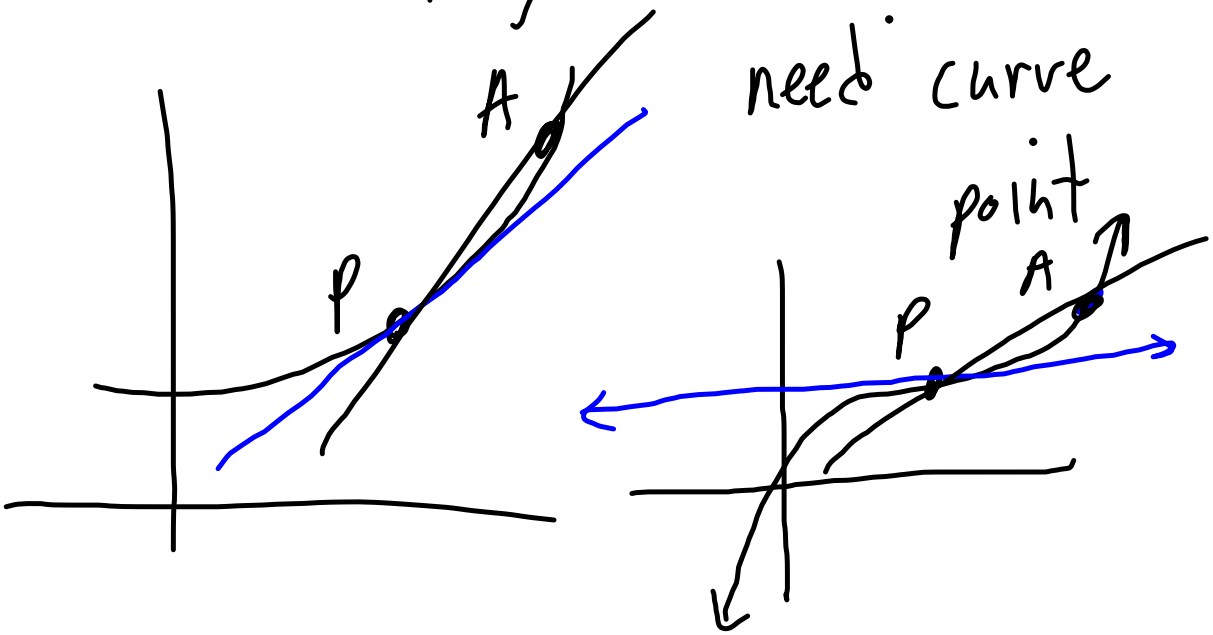
2.4 tangent lines

what is a tangent line?

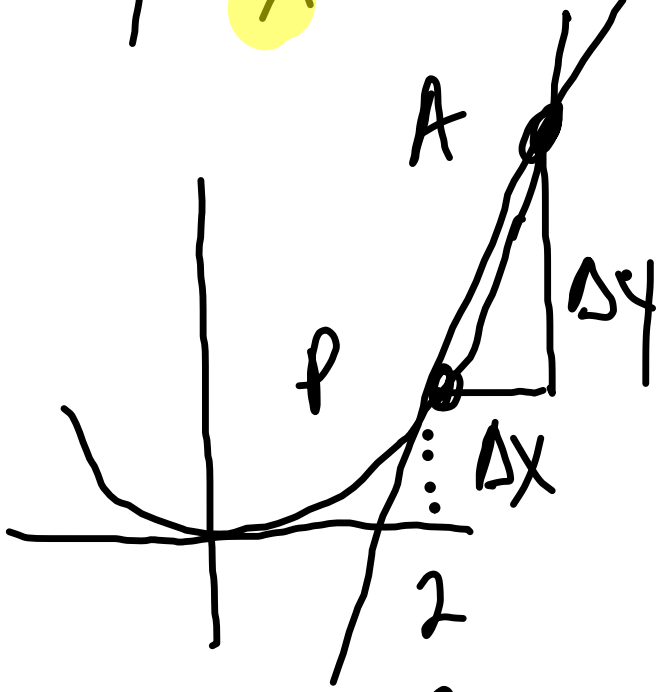
need curve

point



the tangent is the limiting case of the secant as $A \to P$

$y = x^2$ slope of tangent (curve) at $x_1 = 2$



$y_1 = 4$

$x_2 = 2 + \Delta x$

$y_2 = (2 + \Delta x)^2$

$$\lim_{\Delta x \to 0} \left[\frac{y_2 - y_1}{x_2 - x_1} \right] = \text{slope of the tangent}$$

$$\lim_{\Delta x \to 0} \left[\frac{(2+\Delta x)^2 - 4}{2+\Delta x - 2} \right] = \text{slope of tan}$$

$$\lim_{\Delta x \to 0} \frac{4 + 4\Delta x + \Delta x^2 - 4}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x(4 + \Delta x)}{\Delta x} = 4$$

limiting case

slope of tan

#9 $f(x) = x^2$

$x_1 = -2$

$y_1 = 4$

$x_2 = -2 + \Delta x$

$y_2 = (-2+\Delta x)^2$

$$m_{sec} = \frac{(-2+\Delta x)^2 - 4}{-2+\Delta x - -2}$$

$$= \frac{4 - 4\Delta x + \Delta x^2 - 4}{\Delta x}$$

$$= \frac{-4\Delta x + \Delta x^2}{\Delta x}$$

$$= \frac{\Delta x(-4 + \Delta x)}{\Delta x}$$

$$m_{tan} = \lim_{\Delta x \to 0} m_{sec}$$

$$= -4$$

eq. of tan

$m_1 = -\frac{1}{m_2}$

$y = -4(x+2) + 4$

eq of normal $\perp$

$y = \frac{1}{4}(x+2) + 4$

24. how fast = slope of tan

(Inst vel)

$f(x) = 3x^2$

$x_1 = 10$

$y_1 = 300$

$x_2 = 10 + \Delta x$

$y_2 = 3(10+\Delta x)^2$

$$f(x) = \frac{1}{x-1}$$

$$\frac{\frac{1}{(x_1+\Delta x)-1} - \frac{1}{x-1}}{\Delta x}$$